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AN INTRODUCTION TO THE THEORY AND MEASUREMENT
OF DIELECTRIC CONSTANT AND LOSS

by

A. Harry Sharbaugh

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ABSTRACT

The electrical properties of insulation are usually described in terms of a dielectric constant and a loss factor. These quantities are first defined on the basis of macroscopic electrostatic theory and then related to the atomic and molecular structure of the dielectric material. The dependence of the measured values of the dielectric constant and loss upon the prevailing temperature and frequency is discussed. Measurement techniques which are applicable at different frequencies are presented.

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INTRODUCTION

Insulation plays an all-important role in the electrical industry by providing physical support and isolation of conductors from each other and ground, and by serving as an energy storage medium. Through a better understanding of how insulating material behaves in an electric field, engineers may increase the operating efficiency of electrical devices and the chemist has the opportunity of synthesizing new insulation with particular electrical characteristics.

The electrical properties of insulation are usually described in terms of the dielectric constant ϵ. That it should be called a "constant" is somewhat of a misnomer, for, as we shall see, its value may depend on the frequency of the applied voltage and the prevailing temperature. However, ϵ for a given material is independent of the amplitude of the voltage used in its measurement as well as the geometry of the specimen and thus is a characteristic property of the dielectric or insulating material. We shall first regard it as an empirical constant in the development of the macroscopic theory and then examine its dependence upon temperature and frequency on the basis of the atomic and molecular structure of the insulation.

DEFINITION OF AN INSULATOR

We may define what is meant by a dielectric or insulating material on the basis of its resistivity, the resistance in ohm-cm between opposite faces of a one-centimeter cube of the material. On this basis conductors are arbitrarily defined as those materials which have a resistivity less than 10^{-3} ohm-cm and dielectric materials are those which have resistivities greater than 10^{10} ohm-cm. Semiconductors have resistivities which lie between these two limits.

A more sophisticated definition is provided by quantum mechanics.[1] From this viewpoint the possible electronic states of a material are divided into zones in which the energy levels form continuous bands. These zones are separated by intervals of forbidden energy levels as sketched in Fig. 1. The extent to which the lowest permitted
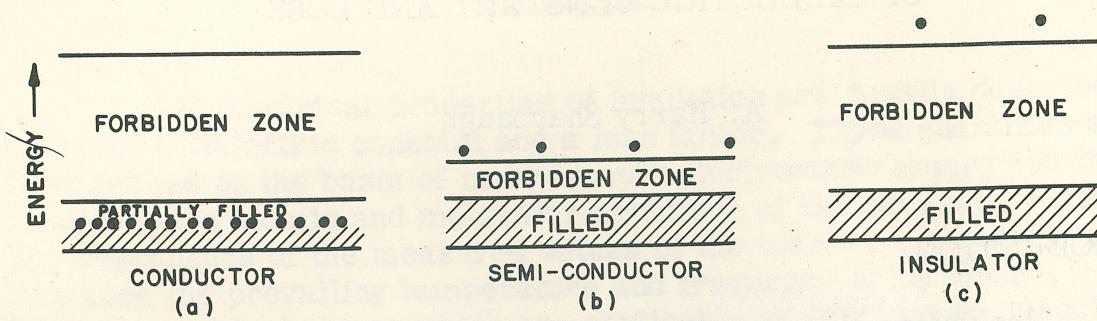
Fig. 1 Quantum mechanical pictures of conductors and insulators.
Conducting electrons at room temperature are represented by
the dots.

zone is filled by electrons is indicated by the shading. In order for an
electron to give rise to an electric current it must be elevated to a
permitted energy level that is unfilled. In a conductor, this is readily
accomplished because the lowest zone is only partially filled and so,
with very little extra energy, an electron may rise to a permitted level
where it can contribute to the conduction current [see Fig. 1(a)]. On the
other extreme, an electron in an insulating material must acquire a
large amount of energy to transcend a large forbidden zone to get into a
permitted conduction band [see Fig. 1(c)]. As the temperature is raised,
more electrons acquire this necessary energy. At room temperature,
however, there are only a few that populate the permitted conduction
band; hence the resistivity is high. Semiconductors [Fig. 1(b)] have
lower resistivities than insulators because the forbidden zone is narrower
and so an electron may more easily acquire the necessary energy to make
the transition into the upper conduction zone.

MACROSCOPIC THEORY

Let us consider the air-filled parallel-plate capacitor of Fig. 2(a)
across which we will apply a d-c potential V. If the plates have an area
A, then $|\sigma A|$ units of charge will appear on each of the two plates, where
σ is the surface charge density. If the applied potential is increased to
2V, the charge density on the plates will be doubled. We thus have the
experimental relation $\sigma A = C_0 V$ where C_0, the capacitance, is the propor-
tionality constant between the charge and the applied voltage. The capaci-
tance of the system thus becomes independent of the applied voltage, i.e.
$C_0 = \sigma A/V$. If the experiment is repeated with a slab of dielectric
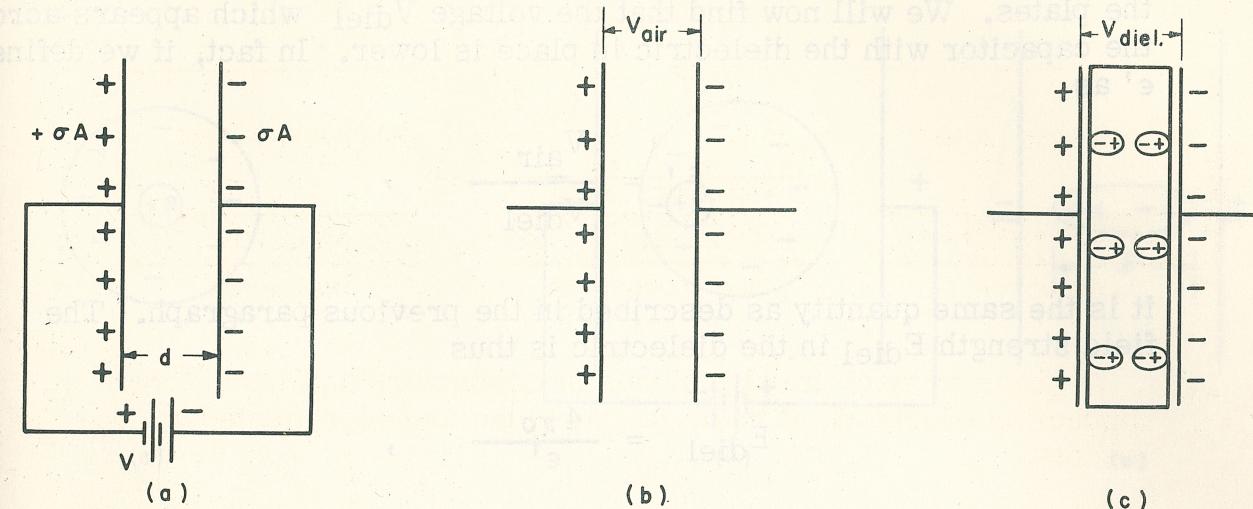
Fig. 2 Polarization of a dielectric material.

material interposed between the plates, we will measure a different and
larger value of capacitance C. The ratio of these two capacitances,
C/C_0, is defined as the dielectric constant ϵ' of the material. With this
definition, the dielectric constant thus becomes independent of the voltage and
geometry of the measuring system and is characterized solely by the
properties of the material between the plates of the capacitor.

We may define ϵ' in another way as follows. Again apply a d-c
voltage, V_{air}, to the air-filled capacitor of Fig. 2(a). The charge $|\sigma A|$
which now appears on each of the plates will give rise to a certain field
strength E_{air} between the plates.[2]

$$E_{air} = 4\pi\sigma$$

By definition

$$E_{air} = \frac{V_{air}}{d} ,$$

Where d is the separation of the two parallel plate electrodes. Let us
now disconnect the d-c voltage so that the charge on the system [Fig. 2(b)]
cannot change and then introduce a slab of dielectric of thickness d between

the plates. We will now find that the voltage V_{diel} which appears across the capacitor with the dielectric in place is lower. In fact, if we define ϵ' as

$$\epsilon' = \frac{V_{air}}{V_{diel}} \quad ,$$

it is the same quantity as described in the previous paragraph. The field strength E_{diel} in the dielectric is thus

$$E_{diel} = \frac{4\pi\sigma}{\epsilon'} \quad ,$$

and this drop in field strength might have been achieved by lowering the surface charge density on each plate an amount $\sigma(1-1/\epsilon')$ by the addition of a compensating charge on the plate. We call the surface density of such a compensating charge the polarization P of the dielectric. It arises from dipole chains which form in the dielectric under the influence of the field. The ends of these chains bind an equal (and opposite in sign) amount of charge on the conducting plate at the surface of the dielectric slab [see Fig. 2(c)]. It is evident that large values of P are consistent with large values for ϵ'. The origin of these dipoles in the dielectric material is the connecting link between the macroscopic theory and the microscopic (molecular) structure of the material. We will discuss this in detail in the next section.

MOLECULAR (MICROSCOPIC) THEORY

Let us consider what happens to the electrically neutral atom (e.g. atomic number 8) pictured in Fig. 3(a) when it is subjected to an electric field. In the absence of a field the centroids of positive and negative charge are coincident and cancel each other. However, under the influence of the field the electrons are attracted and displaced slightly toward the positive electrode with respect to the heavy positive nucleus [Fig. 3(b)]. The atom is now polarized and this kind of polarization is called electronic polarization because we are dealing with a displacement of electrons. Thus, as long as the field is applied, each atom of the material becomes an induced dipole and the centroids of positive and negative charge are no longer coincident. We may represent a polarized atom as two point charges of electricity, q and -q, separated by a distance ℓ and the dipole thus formed is said to have a moment $\mu = q\ell$. [See Fig. 3(c)] For molecules, the charge q will be of the order of unit electronic charge (4×10^{-10} stat. coulombs) and ℓ will be of the order of molecular dimensions (10^{-8} cm); hence molecular dipole moments usually have values in the range $0-5 \times 10^{-18}$ esu.
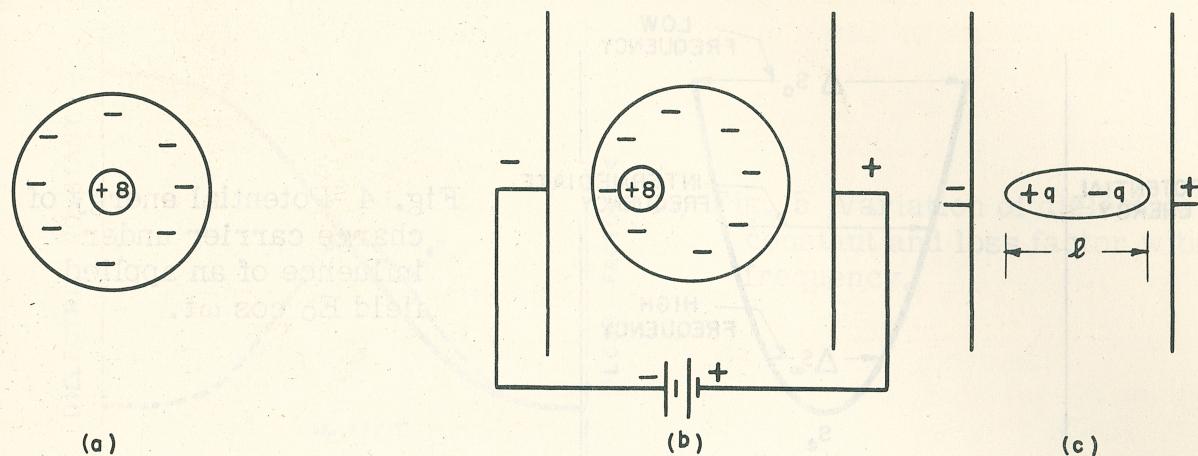

Fig. 3 Formation of an induced dipole.

If an alternating voltage is applied to a dielectric system, it will be necessary for the direction of the polarization to reverse each time the polarity of the field changes. The speed with which this reversal can take place may depend, among other things, upon the mass and size of the charge carrier as well as the viscosity of the dielectric. If the frequency of the applied voltage becomes so high that the polarization does not have time to form, then the contribution of this particular kind of polarization to the net value of the dielectric constant will disappear. This leads to a frequency dependence of the dielectric constant.[3]

A classical analogue may be helpful in visualizing the behavior of a dielectric in an alternating field. Let us consider a weightless carrier of charge e bound by electrostatic forces to a certain equilibrium position S_e (refer to Fig. 4). Its displacement from the equilibrium position by an applied field E will be opposed by two forces: (1) a restoring force proportional to its displacement S, and (2) a frictional force proportional to its velocity v. The differential equation describing this motion will be

$$E e = k_1 v + k_2 S$$

k_1 and k_2 are constants of proportionality. From the discussion in the preceding section, we may conclude that the magnitude of the polarization
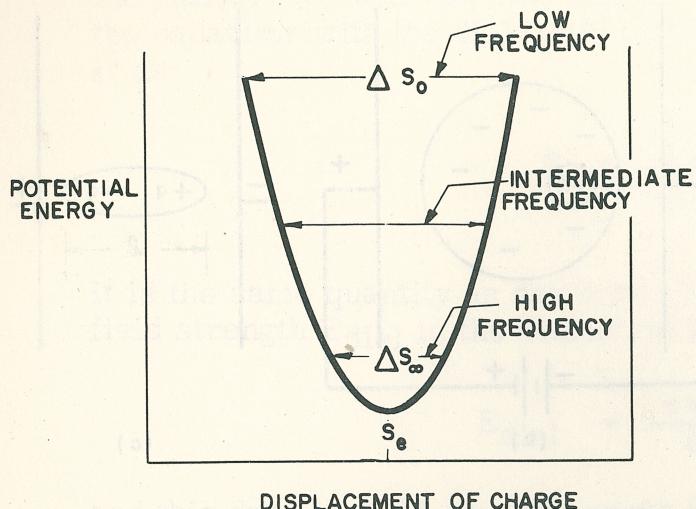
Fig. 4 Potential energy of charge carrier under influence of an applied field $E_0 \cos \omega t$.

(or dielectric constant ϵ') will be proportional, at least qualitatively, to the amount of displacement of the charge carrier.

If the impressed field is d-c, the resultant force displaces the charge over a range of equilibrium values denoted by ΔS_0 of Fig. 4 if the field acts upon the charge for a sufficient time. However, if this d-c field is replaced by an alternating field $E_0 \cos \omega t$, then the total displacement will be ΔS_0 or less and will gradually decrease as the frequency is raised. This will result in a decrease of the dielectric constant (or polarization) as the frequency is increased through the value $\omega_0 = 1/\tau$, where τ is called the relaxation time. Furthermore, the movement of the charge carrier as it is opposed by frictional force will generate an amount of heat per cycle of applied voltage ϵ'' that is proportional to the product of the frictional force $k_1 v$ and the distance S traversed in a cycle, i.e. $\epsilon'' \propto vS$. Now at low frequencies where $\omega << 1/\tau$, ϵ'' will be small because v is small, while at sufficiently high frequencies, $\omega >> 1/\tau$, it is small because S becomes small. The dependence of v and S on frequency is such that at intermediate frequencies ϵ'' passes through a maximum value. This decrease of the dielectric constant and simultaneous appearance of a maximum value of the loss factor as the measuring frequency is increased through some critical frequency is plotted in Fig. 5. This behavior is called anomalous dispersion. We shall see that this characteristic of dielectric materials forms a very important part of dielectric behavior. A complete frequency characteristic of a real dielectric material may be composed of several dispersion regions. They may be widely separated in frequency or overlap. The important
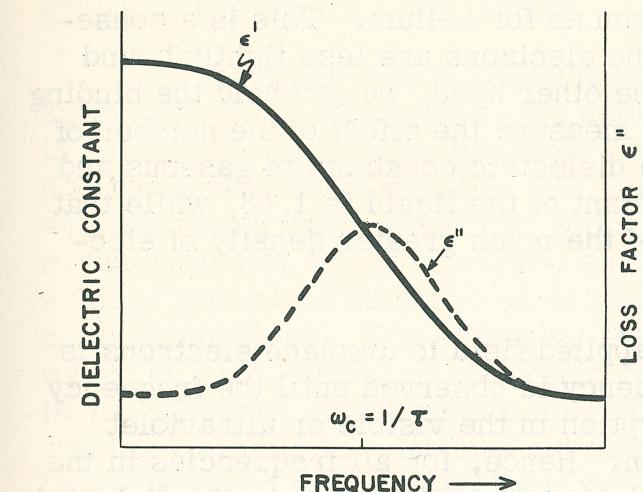
Fig. 5 Variation of dielectric constant and loss factor with frequency.

point is that different kinds of polarization require different amounts of time to form, and so, as the frequency is increased from d.c. to visible light, each type will cease to contribute when it is unable to form during a cycle of the applied field.

TYPES OF POLARIZATION

As a simple example of polarization, we have already briefly discussed electronic polarization, caused by the displacement of electrons with respect to the positive nucleus of an atom. We indicated that there were other kinds of polarization which may form in dielectric materials and we shall now examine electronic polarization in greater detail as well as other important types.

Electronic Polarization

It is reasonable to expect that the amount of electronic polarization should be proportional to the number of bound electrons per cubic centimeter and inversely proportional to the forces which bind the electrons to the positively charged nucleus. That this is true may be illustrated by comparing the polarizations of (or the dielectric constants) of a cc of hydrogen and a cc of helium in the gaseous state. By comparing the same volumes of gas we are comparing the same number of molecules. Both hydrogen and helium have two electrons per molecule, so we are comparing the same number of bound electrons. $(\epsilon'-1)$, the

amount by which the dielectric constant exceeds that of a vacuum, is about four times as large for hydrogen as for helium. This is a consequence of the fact that in hydrogen the electrons are less tightly bound to the nucleus than in helium. On the other hand, we can hold the binding forces of the electrons constant and measure the effect of the number of electrons per cc by comparing the dielectric constants of gaseous and liquid nitrogen. The dielectric constant of the liquid is 1.43, while that of the gas is only 1.0006, because of the much greater density of electrons in the former.

The time required for the applied field to displace electrons is so short that no dependence on frequency is observed until the frequency reaches that corresponding to absorption in the visible or ultraviolet part of the electromagnetic spectrum. Hence, for all frequencies in the power, audio, or radio range, the electronic contribution to the dielectric constant is constant. Maxwell's rule states that $\epsilon' = n^2$, where n is the index of refraction, as long as n and ϵ' are measured at the same frequency. Thus if the square of the index of refraction measured in the optical range is equal to the dielectric constant measured at a low frequency, say at 60 c/s, one may conclude that electronic polarization is the only important kind of polarization. This is approximately the case in benzene where $n_{20°}^2 = (1.501)^2 = 2.253$ and $\epsilon'_{20°} = 2.282$; the latter value is independent of frequency in the electrical range. The small amount by which ϵ' exceeds n^2 is caused by the appearance of another kind of rapidly forming polarization which is called atomic polarization.

Atomic Polarization

As the name implies, this kind of polarization arises from the displacement of atoms in a molecule or of ions in an ionic crystal. In contrast to the movement of the relatively light electron as in electron polarization, this kind of polarization requires the displacement of atoms and so requires more time to form. Consequently, this kind of polarization ceases in the infrared part of the spectrum rather than in the visible as in the preceding case. Usually, the contribution is complete until the frequency is increased to about 10^{10} c/s and stops completely by the time the frequency reaches 10^{14} c/s (near infrared). Atomic polarization has been studied most extensively in alkali halide crystals where it is an important source of polarization. In most organic systems, however, it is relatively unimportant as seen in the benzene example quoted above.

The remaining types of polarization require more time to form than either electronic and atomic polarization and constitute the most important types found in practical insulation systems.

Debye Orientational Polarization

The dipoles which contribute to electronic and atomic polarization owe their existence to the applied electric field. When the field is removed, the dipoles disappear, hence they are often called "induced" polarizations. In contrast to this, orientational polarization is concerned with the lining up of molecules which have a permanent electrical asymmetry built into them. Debye[4] suggested that the high dielectric constant of water and alcohol was caused by permanent dipoles in the molecules themselves. For example, the square of the index of refraction of water is 1.77, while the dielectric constant measured at radio frequencies is 78. Clearly, some kind of contributing polarization other than electronic or atomic has dropped out as the measuring frequency is increased from the radio to the optical spectrum. Debye's postulate opened up a new field of investigation by providing a molecular mechanism to explain dielectric behavior.

Figure 6(a) is a diagrammatic representation of a benzene molecule. In the center is a vector diagram of the individual dipoles which arise because of the different electron affinities of carbon and hydrogen atoms. Since all six individual vectors are equal in length and symmetrically arranged about the center, the resultant dipole moment of the molecule is zero. If, however, a chlorine atom is substituted for one of the hydrogen atoms as in Fig. 6(b), there is no longer complete neutralization of the individual dipoles, and the molecule is no longer in electrical balance. This molecule, chlorobenzene, is said to have a permanent dipole moment.

In a liquid or gas these molecular dipoles are oriented at random and so there is no net polarization. When an electric field is applied, however, there is a tendency for the molecules to align themselves in the electric field and a net polarization results. Such a dipole polarization is thus superimposed upon the two polarizations previously discussed.

When the loss is negligibly small, $\omega \ll 1/\tau$, the value of the dielectric constant ϵ' depends upon the polarizability of the material and how much material is contained in a cubic centimeter, and is independent of the frequency of the measuring voltage. These facts are expressed in the Debye equation[4] as follows:

$$(\epsilon'-1) / (\epsilon'+2) = (4\pi N_1/3) (\alpha_0 + \mu^2/3kT). \qquad (1)$$

The amount of material per cc is given by N_1, the number of molecules per cc α_0 is the measure of the molecular polarizability and is numerically equal to the dipole moment _induced_ in a molecule by unit electric
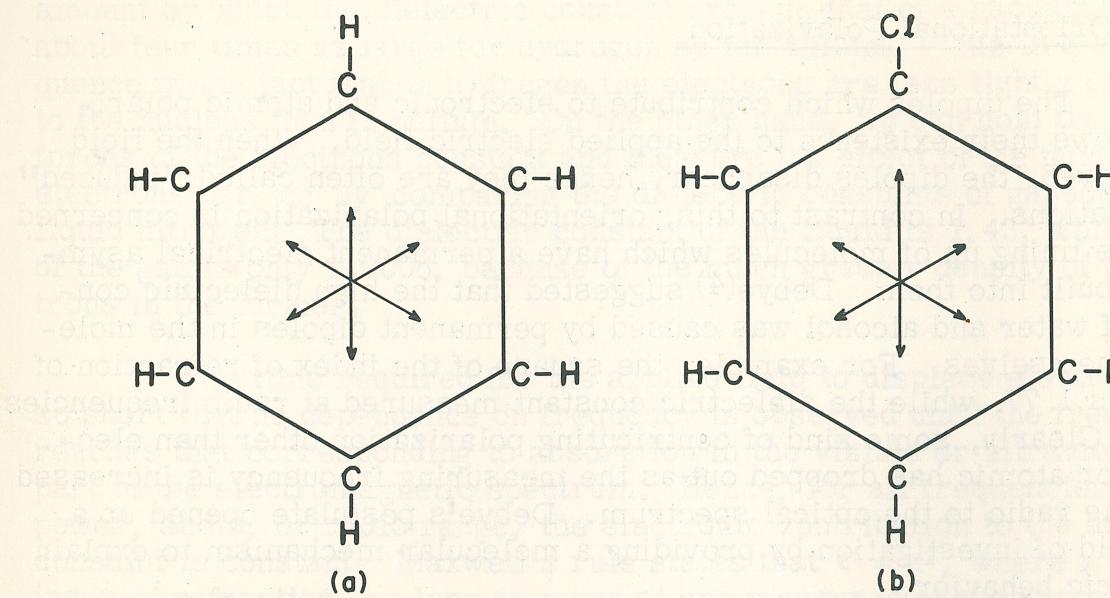
Fig. 6(a) Neutral benzene molecule with zero dipole moment (left); (b) Electrically asymmetrical chlorobenzene molecule with permanent dipole moment (right).

field. The amount of orientational polarizability is taken into account by term $\mu^2/3kT$; k is the gas constant per molecule, 1.38×10^{-16} ergs/degree, T is the absolute temperature, and μ is permanent dipole moment of the molecule. For molecules which have no permanent dipole moment, the second term is therefore zero. N_1 may be calculated from the relation

$$N_1 = 6.02 \times 10^{23} \, d/M,$$

where d is the density in g/cc and M is the molecular weight. One of the triumphs of the Debye theory is the explanation of the large decrease in the dielectric constant with increasing temperature for certain materials. We see in (1) that the only variables on the right hand side are N_1 and T. For nonpolar molecules when $\mu = 0$, N_1 decreases slowly with a temperature increase because of the density change. However, in polar molecules, where the second term may be considerably larger than the first, the decrease will be governed by the $1/T$ dependence.
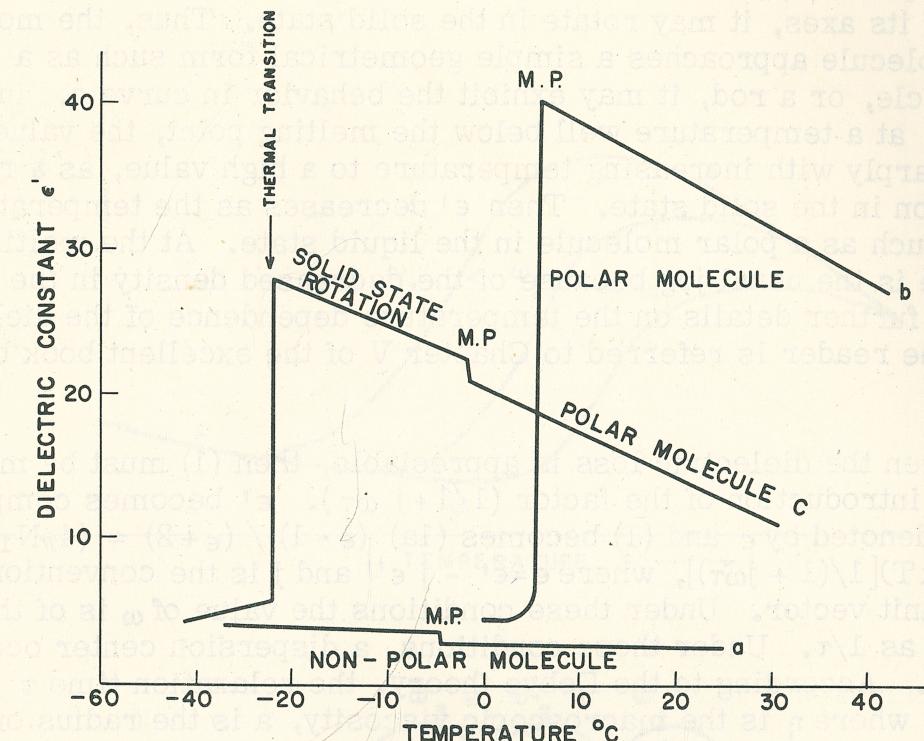

Fig. 7 Examples of the temperature dependence of the dielectric constant. The frequency of measurement is sufficiently low that no dispersion exists: M.P. denotes melting point.

Figure 7 shows the usual temperature dependence of the "static" dielectric constant for several classes of molecules. "Static" dielectric constant is the term used to signify that value of dielectric constant which is measured when the frequency is sufficiently low that no dependence on frequency exists. Curve a shows the behavior of a typical nonpolar molecule. The value of ϵ' is low at all temperatures and equals approximately the square of the index of refraction. As the temperature is increased, the value decreases slowly because of the density change as it affects N_1 in (1). There is a sharp jog in the curve at the melting point because of the smaller density in the liquid state. Curve b shows the typical course of the dielectric constant for a molecule having a large dipole moment. Below the melting point (M.P.) ϵ' is low because the polar molecules are "frozen in" and unable to orient in the field. On melting it rises to a high value when the orientation polarization begins; after which ϵ' decreases rapidly with temperature according to the $1/T$ dependence of

the Debye equation (1). When a polar molecule is symmetrical about at least one of its axes, it may rotate in the solid state. Thus, the more nearly a molecule approaches a simple geometrical form such as a sphere, circle, or a rod, it may exhibit the behavior in curve c. In such cases, at a temperature well below the melting point, the value of ϵ' rises sharply with increasing temperature to a high value, as a result of orientation in the solid state. Then ϵ' decreases as the temperature is raised much as a polar molecule in the liquid state. At the melting point, there is the usual jog because of the decreased density in the liquid state. For further details on the temperature dependence of the dielectric constant, the reader is referred to Chapter V of the excellent book by Smyth.[5]

When the dielectric loss is appreciable, then (1) must be modified by the introduction of the factor $(1/1+j\,\omega\tau)$. ϵ' becomes complex and is now denoted by ϵ and (1) becomes (1a) $(\epsilon-1)/(\epsilon+2) = (4\pi N_1/3)(\alpha_O + \mu^2/3\,kT)[1/(1+j\omega\tau)]$, where $\epsilon = \epsilon' - j\,\epsilon''$ and j is the conventional imaginary unit vector. Under these conditions the value of ω is of the same order as $1/\tau$. Under these conditions, a dispersion center occurs when $\omega\tau = 1$. According to the Debye theory, the relaxation time $\tau = 4\pi\eta\,a^3/kT$, where η is the macroscopic viscosity, a is the radius of an assumed spherical dipole molecule. As the temperature is increased through the value when $\omega\tau = 1$, then the real part of the dielectric constant ϵ' rises from a low value (region of Fig. 8) to the higher static value in region b, where it is free to orient in the field. In region b the value is independent of frequency and the same considerations as discussed in connection with (1) now apply. Increasing the temperature results in decrease in τ both because of the viscosity decreases and the Boltzmann factor kT in the expression for τ. The dispersion occurs at necessarily higher temperatures when the measuring frequency is increased ($\omega_1 > \omega_2 > \omega_3$ of Fig. 8) because of the relation $\omega\tau = 1$. Simultaneously, the loss factor ϵ'' traverses a maximum value with increasing temperature as drawn in Fig. 8 at temperatures where $\omega\tau = 1$. Since ω and τ appear as a product in (1a), increasing τ (by a decrease in temperature) is tantamount to increasing ω. Thus Fig. 8 holds equally well when the temperature abscissa is replaced by ω (with a reversal in sign) and ω_1, ω_2, ω_3, are replaced by T_1, T_2, T_3, respectively, where $T_1 < T_2 < T_3$.

Interfacial Polarization

In a heterogeneous material, an interfacial polarization[6-8] arises because of the accumulation of charge at the interfaces between the phases when they differ from each other in dielectric constant ϵ' and a-c conductivity. Heterogeneity is more common in practical dielectric
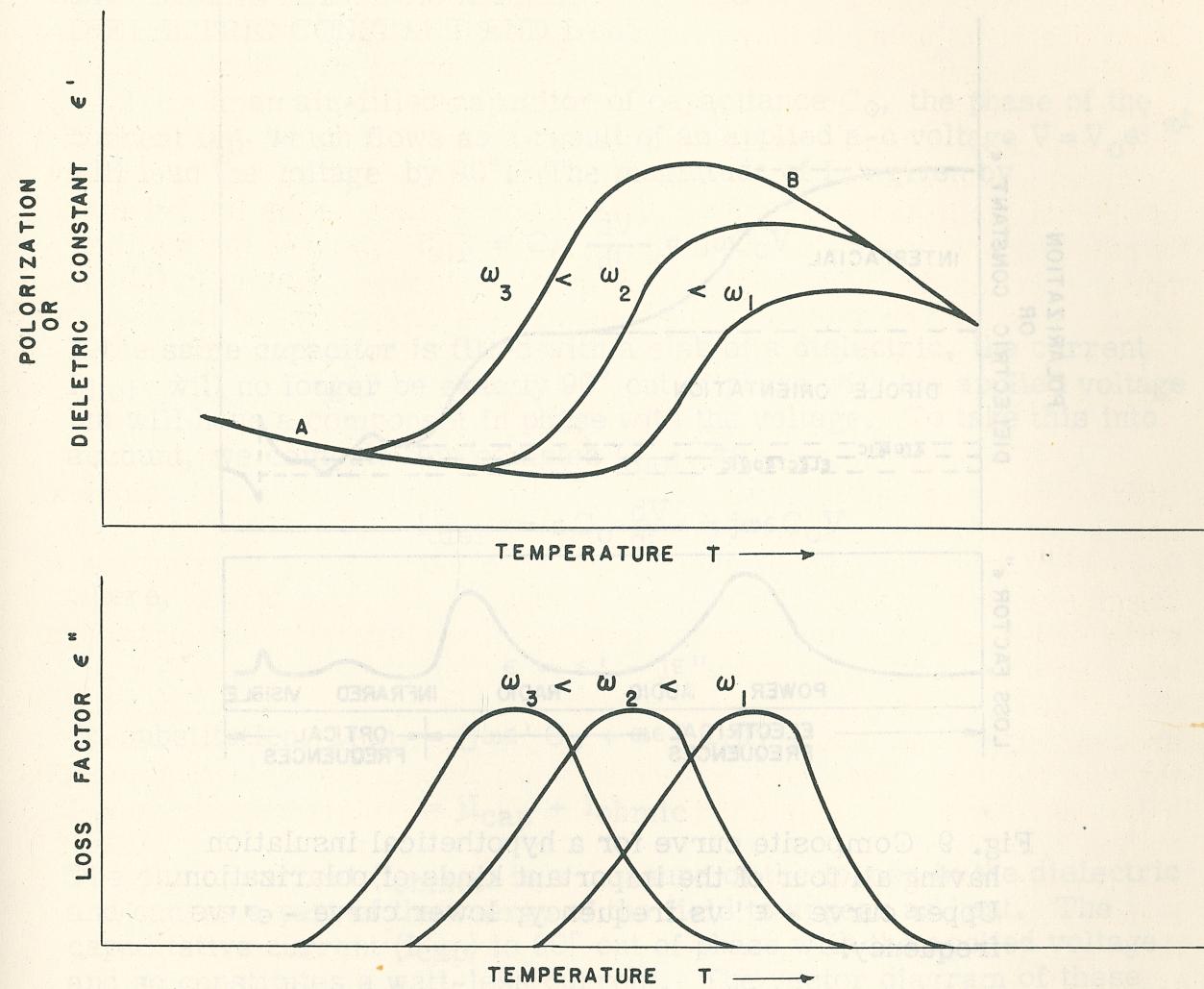

Fig. 8 The temperature dependence of the dielectric constant ϵ' and loss factor ϵ'' when the frequency ω of the measuring voltage is sufficiently high to cause dispersion. $\omega_3 < \omega_2 < \omega_1$.

systems than one might expect. Typical examples are impregnated paper capacitors, laminated plastics, paper with absorbed water, plastics with fillers, and ceramics which have glassy and crystalline phases.

Summary of Polarization Behavior

Each of the different types of polarization may contribute to the dielectric constant depending on how polarizable the insulation is and how many polarizable centers are contained in a cc of material. Figure 9
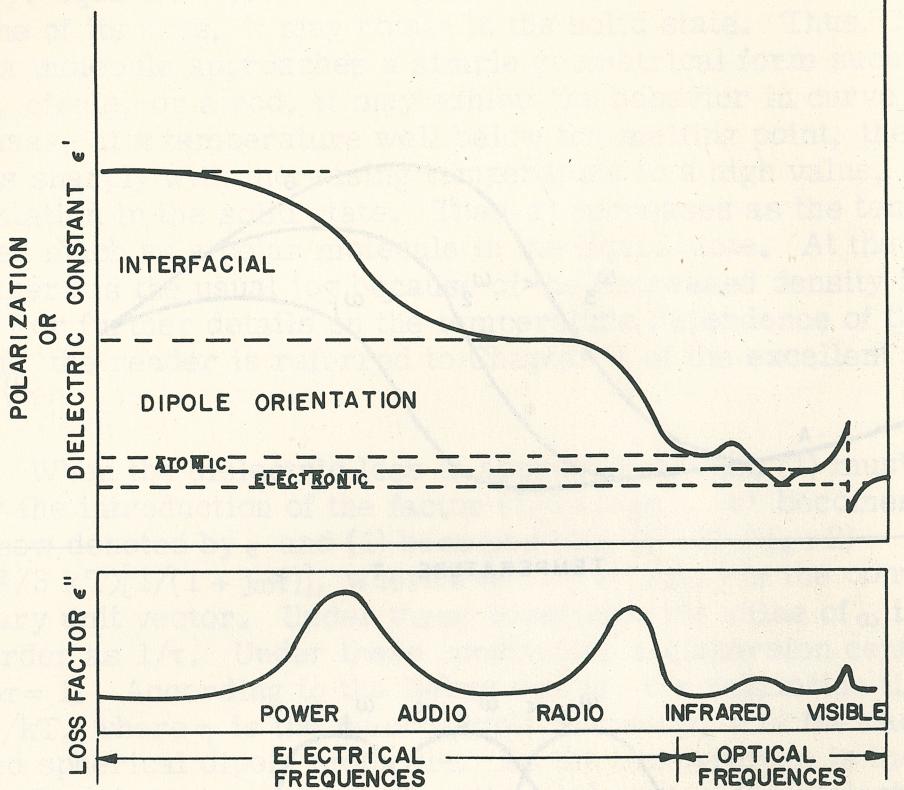
POLARIZATION OR DIELECTRIC CONSTANT ε'

INTERFACIAL

DIPOLE ORIENTATION

ATOMIC

ELECTRONIC

LOSS FACTOR ε''

POWER   AUDIO   RADIO   INFRARED   VISIBLE

ELECTRICAL FREQUENCIES   OPTICAL FREQUENCIES

Fig. 9 Composite curve for a hypothetical insulation
having all four of the important kinds of polarization.
Upper curve - ε' vs frequency; lower curve - ε'' vs
frequency.

shows schematically the decrease in dielectric constant for a hypothetical
material exhibiting all four of the kinds of polarization which have been
discussed. At the lowest frequency all four contribute to a large dielec-
tric constant, and, as the frequency is increased to optical values, one
by one they drop out because of their inability to follow the field reversal.
This curve is thus a superposition of several curves which have the form of
the one shown in Fig. 5. As each polarization ceases to form, it is
accompanied by the appearance of a maximum in the loss factor ε''.

EXPERIMENTAL MEASUREMENT OF THE DIELECTRIC CONSTANT AND LOSS

In an air-filled capacitor of capacitance C_o, the phase of the
current I_{air} which flows as a result of an applied a-c voltage $V = V_o e^{j\omega t}$
will lead the voltage by 90°. The magnitude of I is given by

$$I_{air} = C_o \frac{dV}{dt} = j\omega C_o V$$

If the same capacitor is filled with a slab of a dielectric, the current
I_{diel} will no longer be exactly 90° out of phase with the applied voltage
but will have a component in phase with the voltage. To take this into
account, we compute this complex I_{diel} as

$$I_{diel} = \epsilon C_o \frac{dV}{dt} = j\omega\epsilon C_o V$$

where,

$$\epsilon = \epsilon' - j\epsilon''$$

On substitution, $I_{diel} = (j\omega\epsilon' C_o + \omega\epsilon'' C_o)V$

$$= jI_{cap} + I_{ohmic}$$

The ohmic current (I_{ohmic}) is a measure of the losses in the dielectric
and causes a part of the energy of the field to appear as heat. The
capacitive current (I_{cap}) is 90° out of phase with the applied voltage
and so constitutes a watt-less current. The vector diagram of these
currents is shown in Fig. 10. The angle δ between I_{cap} and I_{diel} is
called the loss angle and the tangent of this angle

$$\tan \delta = \frac{\text{ohmic current}}{\text{capacitive current}} = \frac{\epsilon''}{\epsilon'}$$

Thus tan δ becomes a figure of merit for a dielectric; small values indi-
cate either small losses and (or) large energy storage (large capacitance).
When tan δ < 0.1

$$\tan \delta \simeq \sin \delta = \cos \phi = P.F.$$

where P.F. denotes the power factor of the dielectric. Thus the power
factor and tan δ are equal only for low loss dielectrics where this
approximation holds. In Table I are listed some values for these dielec-
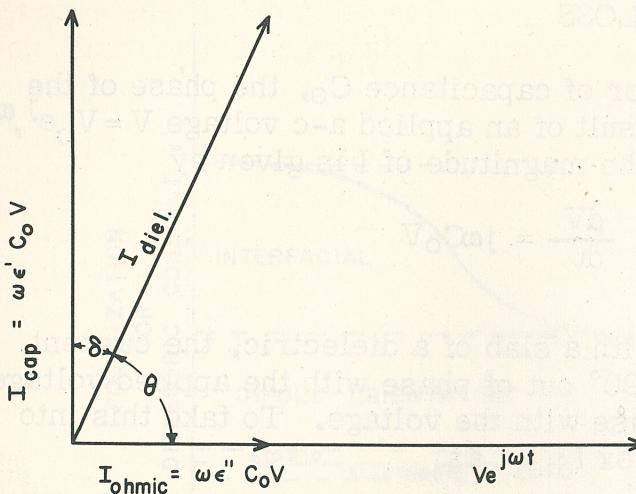tric parameters to give an idea of magnitude.

Fig. 10 Vector diagram of the current through a dielectric.

TABLE I

Dielectric Properties of Some Materials at 25°C and 60 c/s

Material	ϵ'	ϵ''	Loss tangent (tan δ)	Power Factor (cos φ)
Liquids				
Pyranol	5.3	0.005	0.001	0.001
Mineral oil	2.2	0.002	0.001	0.001
Dibutyl sebacate	4.5	0.040	0.010	0.010
Water	78.	23,000	300	1.000
Solids				
Polystyrene	2.55	.0006	.0003	.0003
Plexiglas	3.40	.204	.060	.060
Gases				
Air	1.00536	0	0	0
Carbon dioxide	1.000920	0	0	0

Let us next examine the relationship between the complex dielectric constant ϵ and the a-c conductivity.[2] For conductors, the conductivity γ is defined as the factor by which the voltage gradient E must be multiplied to give the current density I

$$I = \gamma E$$

or the factor by which the square of the voltage gradient must be multiplied to give the heat W generated per second in a centimeter cube of the material,

$$W = IE = \gamma E^2$$

Regardless of the composition of a <u>conductor</u>, the heat produced per second by a given applied voltage is proportional to the current. This proportionality between current and heat produced does not obtain in insulators, because the ratio between current flowing and heat developed varies with the composition and kind of insulation. We have seen in the preceding discussion that the total current in the dielectric is composed of an ohmic and a capacitative part and that only the former leads to heat generation. Since tan δ, the ratio of ohmic to capacitative current, varies from material to material as well as with frequency, it comes as no surprise that this proportionality between heat produced and total current does not hold for dielectric materials. Some dielectrics have an appreciable conductivity with d-c voltages because of the free ions present. This leads to the formation of Joule heat just as in a conductor, and this heat is added to that formed by a pure a-c voltage mechanism. In these cases, the dielectric loss is thus proportional to the <u>total</u> measured a-c conductivity minus the d-c conductivity.

We are now ready to develop the mathematical relations between the complex dielectric constant ϵ and the quantities which can be measured experimentally. Consider a parallel plate capacitor filled with dielectric and having a plate separation d centimeters and a plate area of A square centimeters. If a voltage V is applied to the plates, a charge σ per square centimeter will appear on either of the plates and a polarization P will appear in the dielectric. The conductivity γ will be defined for the dielectric by the same relation as for conductors

$$I = \gamma E = \frac{d \sigma}{dt} , \qquad (2)$$

where E = V/d. The charge density σ can be calculated from the laws of electrostatics[3] by the following relations:

$$D = \epsilon E, \text{ and}$$

$$D = 4 \pi \sigma$$

The quantity D is called the displacement vector. Hence (2) becomes

$$\frac{d\sigma}{dt} = \gamma E = \frac{1}{4\pi}\frac{dD}{dt} = \frac{\epsilon}{4\pi d}\frac{dV}{dt} \cdot \text{(esu)} \qquad (3)$$

When the applied potential is alternating, V may be expressed as the real part of $V = V_o e^{j\omega t}$ where V_o is the amplitude. Using the relation

$$\epsilon = \epsilon' - j\,\epsilon'' \qquad ,$$

the current density in the dielectric is then

$$\frac{d\sigma}{dt} = j\omega(\epsilon' - j\epsilon'')\ \frac{V_o}{4\pi d}\ e^{j\omega t} \qquad (4)$$

$$= \frac{\epsilon''\omega}{4\pi} + j\,\frac{\epsilon'\omega}{4\pi}\ E_o e^{j\omega t} \qquad (4a)$$

$$= (\gamma' + j\gamma'')\ E_o e^{j\omega t} \qquad , \qquad (4b)$$

where $\gamma' = \epsilon''\omega/4\pi$ and $\gamma'' = \epsilon'\omega/4\pi$. Thus γ of (3) is the complex conductivity

$$\gamma = \gamma' + j\,\gamma''$$

The experimentally measured quantity of a dielectric material is the impedance Z (or admittance Y) of a capacitor containing the dielectric in question. The admittance Y is given by the expression

$$Y = 1/R_P + j\omega C_P = G_P + j\omega C_P \qquad (4c)$$

for the equivalent parallel circuit of the dielectric given in Fig. 11. The current I in amperes is given by

$$YV = (G_P + j\omega C_P)\,V \qquad \text{(mhos, farad, volts)}. \quad (5)$$

To convert this current into esu and into current density as used in (4), we divide (5) by the area of the plate A and multiply by 0.9×10^{12} which is the ratio of the farad to the electrostatic unit of capacitance and also the mho to the electrostatic unit of conductance. Thus

$$d\sigma/dt = (0.9 \times 10^{12}/A)(G_P + j\omega C_P)\ V_o e^{j\omega t} \qquad . \quad (6)$$
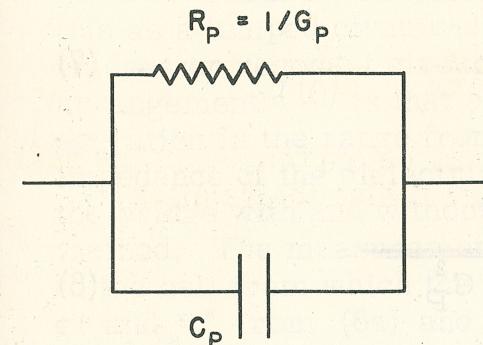

$R_P = 1/G_P$

C_P

Fig. 11 Equivalent circuit of a dielectric sample. Both R_P and C_P are functions of the frequency.

As a part of the experimental procedure, the capacitance C_o of the empty capacitor in farads is either measured or calculated by the expression

$$C_o = \frac{A}{4\pi d \times 0.9 \times 10^{12}}$$

We may now compare (6) with (4), (4a), and (4b) to obtain γ', ϵ'', and ϵ' in terms of the quantities G_P and C_P as measured on a bridge or resonant circuit. Thus,

$$\epsilon' = C_P/C_o \qquad , \quad (6a)$$

$$\epsilon'' = G_P/\omega C_o \qquad , \quad (6b)$$

$$\gamma' = G_P/4\pi C_o, \text{ and}$$

$$\gamma' = \epsilon''\omega/4\pi = \epsilon''\,f/2 \qquad . \quad (6c)$$

In these equations ϵ', ϵ'', and γ' are expressed in esu, while C_P and C_o are expressed in farads and G_P in mhos. ω is the measuring frequency in ra/sec, while f is in cycles/sec. To calculate γ' in ohm^{-1} cm^{-1}, (6c) becomes

$$\gamma' = \epsilon''\,f/1.80 \times 10^{12}$$

We have previously defined $\tan \delta = \epsilon''/\epsilon'$. In terms of the directly measurable quantities, then

$$\tan \delta = G_P / \omega C_P \qquad . \quad (7)$$

Similarly, the power factor (P.F.) becomes

$$\text{P.F.} = \cos \phi = \frac{G_P}{\sqrt{G_P^2 + \omega^2 C_P^2}} \qquad . \quad (8)$$

When $\tan \delta \leqslant 0.1$ (i.e., $G_P \leqslant 0.1 \omega C_P$), (8) reduces to (7) and the power factor becomes identical to the loss tangent in a manner consistent with our earlier discussion on the basis of the vector diagram of the current in Fig. 10.

In (4b) we note that the component of the instantaneous current which is in phase with the applied field is

$$\gamma' E_O e^{j\omega t}$$

To obtain the instantaneous heat developed per sec (instantaneous power) we multiply this current by the applied field $E_O \cos \omega t$. By integration over a whole number of half periods, we obtain the mean heat developed per sec (mean power) $\overline{W}$.

$$\overline{W} = \gamma' \left(\frac{E_O}{\sqrt{2}}\right)^2 = \frac{\epsilon'' \omega}{4\pi} \left(\frac{E_O}{\sqrt{2}}\right)^2 , \quad \left(\text{ergs/sec/cc}, \quad \text{peak stat volts/cm, ra/sec}\right)$$

$$\text{and} \qquad = 0.555 \times 10^{-12} f \epsilon'' \left(\frac{E_O}{\sqrt{2}}\right)^2 \left(\text{watts/cc}, \quad \text{peak volts/cm, cycle/sec}\right)$$

$$\overline{W} \text{ (per cycle)} = \frac{\epsilon''}{2} \left(\frac{E_O}{\sqrt{2}}\right)^2 . \quad \left(\text{ergs/cycle/cc}, \quad \text{peak stat volts/cm}\right) .$$

Because of space limitations, it will not be possible to describe all the various types of experimental gear used in the measurement of the dielectric constant and loss. For greater detail, reference may be made to a book in which a chapter on this subject has been written by the author and S. Roberts.[9] Apparatus will be described, however, which will serve to illustrate the general principles which are involved. The choice of a particular method will depend on individual factors such as (1) the temperature and frequency at which measurements are desired (2) the physical state of the insulation--gas, liquid, or solid (3) how much material is available and the practicability of shaping it into a specimen and (4) special techniques which are necessary for making measurements on powders or thin sheets of material.

From d-c to about 10^8 c/s, we may regard our slab of dielectric as a lumped circuit element whose impedance can be measured with bridges or resonant circuit devices. The most versatile of the bridge arrangements[10] is that of the Schering circuit which is suitable for operation in the range from about 60 c/s to about 3×10^5 c/s. The impedance of the dielectric-filled cell is usually obtained by balancing the bridge with and without the cell connected i.e., the substitution method. The measurement gives the apparent resistance and capacitance of the cell from which the R_P and C_P of (4c) may be calculated to obtain ϵ' and ϵ'' from (6a) and (6b). At higher frequencies ($0.5 \times 10^6 - 40 \times 10^6$ c/s) the "Twin-T" circuit eliminates troublesome stray capacitance and inductance effects in the coupling transformer of the Schering circuit. Because the self-inductance of the leads to the measuring cell becomes important in this frequency range, specially designed cells are necessary.[11]

A guard ring electrode is advantageous in a sample cell because it eliminates surface leakage current and avoids "fringing" of the capacitance at the edges of the electrode. Corrections for fringing may be employed[12] but it is better to eliminate the need for them and calculate the capacitance from the cell geometry. Both of the bridge circuits described above may be used with a guard ring, but the additional circuit makes the bridge balancing procedure very tedious. A bridge which permits the use of a guard electrode which is automatically balanced has been described[13, 14] and may be used up to a frequency of 5×10^6 c/s.

In the resonance method of measuring dielectric constant and loss, the dielectric-filled cell is connected in parallel with an inductor L and a standard capacitor. This circuit is loosely coupled (inductively or capacitatively) to a generator and a high impedance voltmeter is connected across the cell. The standard capacitor is then adjusted until the voltmeter indicates a maximum value and the circuit is said to be in resonance. This will occur when

$$f_O = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$

f_O is frequency of the generator in c/s, L is the inductance in henries, and C is the capacitance in farads. The cell is then disconnected from the circuit and the increase of capacitance which is necessary to restore resonance is a measure of the capacitance C_P of the dielectric-filled cell. This procedure is repeated to obtain C_O, the capacitance of the empty cell, and ϵ' may be calculated from (6a). The loss is obtained

by measuring the sharpness of resonance or Q of the circuit. $Q = f_0/2\Delta f$, where Δf is the deviation from the resonant frequency f_0 required to reduce the voltage to $1/\sqrt{2}$ of its maximum value. The loss tangent of the sample is given by

$$\tan \delta = 1/Q - 1/Q_0$$

where Q denotes the value measured with sample, and Q_0 is the value for the circuit without the dielectric. Resonant circuit techniques are generally useful in the range of frequencies from 10^4 to about 10^8 c/s. Many variations in this basic procedure have been described in the literature to meet specific requirements.[15, 16]

Between 3×10^8 and 10^{10} c/s, the dimensions of the inductor and the sample become the same order as one-quarter of the operating wavelength, and it is necessary to use resonant cavities. Re-entrant cavities have been used[17] successfully in the range $3 \times 10^8 \times 6 \times 10^8$ c/s. The general procedure is very similar to that employed with the resonant circuits at lower frequencies. Thus, the resonant frequency and the Q of the cavity is measured with and without the sample. From these observations ϵ' and ϵ'' are calculated.

In the range 3×10^9 to 6×10^{10} c/s, the re-entrant cavity is replaced by the waveguide resonator. In this range, a special equipment is required at each frequency and the samples have to be tailor-made to fit the apparatus. Although the appearance of the resonator is vastly different from the lower frequency equipments, the general principle of the method still remains the same. However, the voltage is no longer constant throughout the specimen and distributed circuit concepts must be employed. For a discussion of microwave measurement procedures, the reader is referred to a publication by R.M. Redheffer.[18]

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TECHNICAL INFORMATION SERIES

Title Page

AUTHOR	SUBJECT CLASSIFICATION	NO. 57-RL-1787
Sharbaugh, A.H.	dielectric phenomena	DATE September 1957

TITLE	
An Introduction to the Theory and Measurement of Dielectric Constant and Loss	15-5701-090

ABSTRACT

     The electrical properties of insulation are usually described in terms of a dielectric constant and a loss factor. These quantities are first defined on the basis of macroscopic electrostatic theory and then related to the atomic and molecular structure of the dielectric

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CONCLUSIONS

material. The dependence of the measured values of the dielectric constant and loss upon the prevailing temperature and frequency is discussed. Measurement techniques which are applicable at different frequencies are presented.

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SECTION: Physical Chemistry

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What is resistance to Lightning Puncture of Lapp *Hi-F*?

THE PROBLEM: Utility, contemplating purchase of large quantity of suspension insulators, requested complete test data on Lapp *Hi-F* suspension insulators, including results of steep-wave impulse tests.

THE TEST: No standards exist for steep-wave tests on insulators; (the most severe standard we could find is one specified for power transformers calling for a 1,000-kv-per-microsecond rate of rise—considered to be more severe than natural lightning). To be sure of adequate margins of safety in the *Hi-F* design, puncture resistance of its "compact" head had been a matter of particular attention. Design tests of strings of eight *Hi-F* units had showed no insulator failures under repeated flashover using a wave of 3,500-kv-per-microsecond rise. However, at suggestion of the inquiring utility, we undertook impulse flashover tests with rates of rise up to 6,000-kv-per-microsecond.

THE FINDING: Three Lapp No. 9000 *Hi-F* suspension insulators, taken from stock at random, were tested individually, voltage measured across the test specimens being recorded oscillographically. One specimen received 25 applications of flashover voltage at the 6,000-kv-per-microsecond maximum rate of rise; each of the other two specimens received ten such flashovers. In no case was any unit punctured or damaged other than for typical surface marking—etched paths across the glaze—produced by the high-energy arc. (Such steep-wave arcs do not flash clear of the insulator, as is characteristic of lower-speed impulses, but hug tightly to the surface.) Another *Hi-F* suspension used in calibrating the equipment prior to the test, was subjected to 75 flashover shots at ultra-steep-waves ranging from 2,000 to 4,500-kv-per-microsecond; no puncture or damage occurred.

THE CONCLUSION: No impairment of strength or electrical performance of the Lapp *Hi-F* suspension insulator resulted from a test which is technically believed to be many times more severe than natural lightning flashover. Our inquiring customer believes this to be a convincing demonstration of the ability of Lapp *Hi-F* to resist lightning puncture—Lapp *Hi-F* now has full approval of this utility. Don't you think Lapp *Hi-F* meets your requirements, too?

Lapp Insulator Co., Inc., LeRoy, New York

60.5-kv/cm

0.1 µs/cm
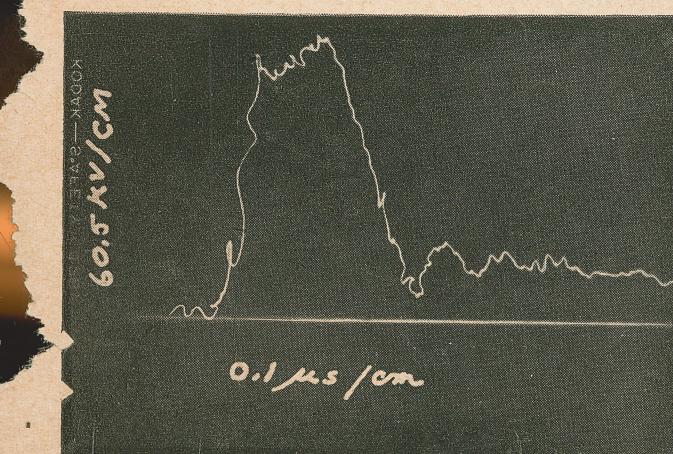
Oscillograph recording of one steep-wave flashover of Lapp Hi-F suspension insulator. Peak voltage reached was 230 kv.

Lapp *Hi-F*